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Circumventing the eta problem in building an inflationary model in string theory

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The eta problem is one of the most significant obstacles to building a successful inflationary model in string theory. Planck mass suppressed corrections to the inflaton potential generally lead to inflaton masses of order the Hubble scale and generate contributions of order unity to the η slow roll parameter rendering prolonged slow roll inflation impossible. We demonstrate the severity of this problem in the context of brane anti-brane inflation in a warped throat of a Calabi-Yau flux compactification with all phenomenologically dangerous moduli stabilized. Using numerical solutions we show that the eta problem can be avoided in scenarios where the inflaton is non-minimally coupled to gravity and has Dirac-Born-Infeld (DBI) kinetic term. We show that the resulting cosmic microwave background (CMB) observables such as measures of non-gaussianites can, in principle, serve as a probe of scalar-gravity couplings.

1. INTRODUCTION

It has proven notoriously difficult to connect the inflationary universe paradigm [1] with fundamental particle physics. However, due to the recent advances in our understandings of flux compactifications, moduli stabilization and various compactification effects [2–4], string theory is beginning to provide relatively concrete settings for inflationary model building [5]. One of the most severe difficulties, preventing the construction of successful models, is the eta problem. For a canonically normalized inflaton φ of mass m, the slow roll parameter η as a function of the Hubble paramter H is: $\eta = M_{pl}^2 \frac{V''}{V} = \frac{m^2}{3H^2}$, where prime denotes differentiation with respect to φ. Therefore, an inflaton mass of order the Hubble scale is equivalent to $\eta \sim \mathcal{O}(1)$. Successful slow roll inflation yielding approximately 60 e-foldings of inflation requires $m^2 \ll H^2$. In string theory, Planck suppressed corrections to the inflaton potential of the form

$$\hat{\mathcal{O}}_4 \frac{\varphi^2}{M_{pl}^2}\,, \tag{1}$$

for some operator of dimension four $\hat{\mathcal{O}}_4$, generically lead to $\eta \sim 1$, when $\left\langle \hat{\mathcal{O}}_4 \right\rangle \sim V$ [6].

In this paper, we explicitly show that the eta problem can be avoided in models of relativistic brane inflation where the noncanonical nature of the inflaton kinetic term is significant. We present numerical solutions that demonstrate this effect, providing concrete realizations of the mechanism envisioned by Silverstein and Tong in [7]. We are primarily concerned with a contribution to the mass term coming from a non-minimally coupled inflaton field $\xi\varphi^2 R$, although we present a general discussion of problematic mass terms in Section 3. We point out the interesting fact that gravitational couplings in DBI inflation models alter observable quantities and the sound speed. This can lead to new observational signatures in the cosmic microwave background (CMB) radiation.

A particularly well understood example of the eta problem occurs in brane inflation, [8, 9], where the inflaton is a (slow-roll canonical) scalar field $\varphi = \sqrt{T_3} r$ parameterizing the position r of a D3-brane moving towards an anti-D3-brane in a warped throat region of a Calabi-Yau flux compactification. Here, $T_3 = \left((2\pi)^3 g_s \alpha'^2\right)^{-1}$ is the brane tension and g_s is the string coupling. Inflation ends when the proper distance between the brane and the anti-brane reaches the string length $\ell_s \sim \sqrt{\alpha'}$ and tachyon condensation occurs. The effective inflaton potential is usually written in the form

$$V(\varphi) = V_{D3/\overline{D3}} + H^2\varphi^2 + \Delta V(\varphi)\,. \tag{2}$$

The $V_{D3/\overline{D3}}$ is the attractive Coulomb potential between the brane and the anti-brane. This is extremely flat for large φ. The second term is the mass term $H^2\varphi^2$ associated with (1) and generically ruins slow-roll inflation. The mass term is related to moduli stabilization effects [4] and was calculated in [9]. The final term ΔV includes all possible additional perturbative and non-perturbative corrections.

A recent important result [4] demonstrates that under specific conditions a non-chiral dimension two CFT operator $\hat{\mathcal{O}}_2$ interacting with a bulk moduli field X induces

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a negative contribution to the radial potential:

$$\Delta V(\varphi) = -c\, a_0^4\, T_3 \left(\frac{\varphi}{\varphi_{UV}}\right)^2 , \qquad (3)$$

which may be tuned to cancel the problematic second term in (2). In the above, $c > 0$, a_0 is related to the minimal warp factor in the throat and φ_{UV} relates to the ultra-violet (UV) cutoff of the throat geometry [4].

While this result is a significant step towards addressing the eta problem in the context of the specific model outlined above, we will focus on another possibility. In D-brane inflation models, the inflaton kinetic term is of the Dirac-Born-Infeld (DBI) form [10], containing an infinite sum of higher derivative kinetic operators. The nonstandard form of the kinetic Lagrangian imposes an effective speed limit on the inflaton field allowing for a new form of slow-roll inflation even in very steep potentials [7]. Here, we explicitly demonstrate the ability of the DBI mechanism to circumvent the eta problem by providing numerical solutions giving rise to at least 60 e–foldings of inflation while in the presence of a conformally coupled mass term [1].

2. COMPACTIFICATION DATA

The setting for our scenario is a flux compactification of Type IIB string theory on an orientifold of a Calabi-Yau threefold [12]. The line element is

$$ds^2 = h^{-1/2}(y) g_{\mu\nu} dx^\mu dx^\nu + h^{1/2}(y) g_{mn} dy^m dy^n , \quad (4)$$

where $h(y)$ is the harmonic function of the manifold, $g_{\mu\nu}$ is the four-dimensional metric and g_{mn} is the metric on the internal space. We assume the internal space has a conical throat where the metric may be written locally as $g_{mn}dy^m dy^n = dr^2 + r^2 ds^2_{X_5}$, where X_5 is some Sasaki-Einstein space (for example S^5 or $T^{(1,1)}$ which is topologically $S^2 \times S^3$). For sufficient flux, the throat may be strongly warped and this warping is captured in the form of the harmonic function (or "warp-factor") $h(y)$. The throat is smoothly glued to the compact space and the complex structure moduli are fixed as in [13]. The setup is assumed to have a single Kähler modulus ρ. The modulus is stabilized via strong gauge dynamics on the world-volume of n supersymmetrically wrapped D7-branes on a four-cycle, which generate a nonperturbative contribution to the superpotential, $W_{\rm np} = A(z_\alpha)\exp[-2\pi\rho/n]$ [14]. The prefactor A depends on the three complex D3-brane coordinates $z_\alpha = \{z_1, z_2, z_3\}$. The warped throat is supported by a background D3-brane charge $N \gg 1$ and is locally well approximated by the near horizon D3 brane geometry[2] $AdS_5 \times S^5$, with warp factor h, and radius of curvature, $\Re$, given by:

$$h(y) = \left(\frac{\Re}{r}\right)^4 , \qquad \Re^4 = 4\pi g_{\rm s} N \alpha'^2 . \qquad (5)$$

In the following discussion it will be useful to define the function f, related to the harmonic function of the warped compactification (5), by, $f(\varphi)^{-1} = T_3 h(\varphi)^{-1}$. We also introduce the parameter

$$\lambda \equiv T_3\, \Re^4 = \frac{N}{2\pi^2} , \qquad (6)$$

so that $f(\varphi) = \lambda/\varphi^4$.

A major difficulty in inflationary model building in the above setting is the upper bound on the inflaton field range [16, 17]:

$$\frac{\Delta\varphi}{M_{pl}} \leq \frac{2}{\sqrt{N}} , \qquad (7)$$

where this particular expression for the bound was derived by Baumann and McAllister (BM bound) in [17].

For the purpose of this demonstration, we consider the so-called *delicate model* of slow-roll brane inflation[3] [3]. This model hinges on the calculation of a one-loop correction to the volume-stabilizing nonperturbative superpotential [2], which (for fine-tuned values of the microphysical parameters) leads to an effective approximate inflection point potential for the inflaton. The potential is well approximated by

$$V(\varphi) = V_0 \left(1 + \lambda_1(\varphi - \varphi_\star) + \frac{1}{3!}\lambda_2(\varphi - \varphi_\star)^3\right) , \quad (8)$$

where $\varphi_\star$ is the location of the inflection point. After sufficient tuning of parameters, it is possible to construct a viable slow-roll inflationary model[4]. We consider this particular form for the potential because inflection point potentials are a common feature of many string inflation models [20], although our results are applicable to more general potentials.

Our strategy will be to numerically construct a successful model of slow-roll brane inflation using the above setup. We then demonstrate that this inflationary solution is lost if we include the effects of a particular dimension-six Planck suppressed operator correction term $\hat{\mathcal{O}}_6$ of the form (1). Finally, we demonstrate that inflation can be salvaged if one appropriately takes into account the DBI

[1] An interesting inflationary model driven by a non-minimally coupled Standard Model Higgs field appeared in [11].

[2] Here we integrate out the angular degrees of freedom focusing on the radial motion of the brane as in [3]; however, in general, angular structure may play an interesting role [15].

[3] Our results are applicable to more generic situations for example the recently constructed models of [4].

[4] Recent work suggests the overall tunings may be less severe than initially anticipated [18, 19].

nature of the inflaton kinetic term. Note that we are not concerned with building inflationary solutions that conform to the most recent observations, only in building solutions that give rise to more than $\mathcal{N} = 60$ e–foldings of inflation and are consistent with all known compactification constraints.

3. PHENOMENOLOGY OF PROBLEMATIC MASS TERMS

Multiple sources can ultimately contribute to Hubble mass correction terms of the form $H^2\varphi^2$. We devote this Section to a rudimentary discussion of this potentially confusing topic. In effective field theory (EFT), one expects a gravitational coupling term, $\xi R\varphi^2$, to appear in the scalar action due to the nature of quantum fields in a curved spacetime. The term is renormalizable by power counting arguments and therefore must be included in the curved spacetime scalar field Lagrangian. Even if it is not present in the bare Lagrangian it will be generated by quantum effects: minimal coupling ($\xi = 0$) is non-generic and unstable to quantum corrections. Moreover, the term has been explicitly calculated in the case of a Dp-brane probe in a negatively curved Einstein space with conformal boundary with a curved boundary metric (such as asymptotically AdS_{p+2}) [21, 22]. In this case, the case relevant for brane anti-brane inflation, ξ has the conformal value[5] $\xi = 1/6$.

During inflation, the Hubble parameter is essentially constant ($\dot{H} \simeq 0$). The Ricci scalar in a cosmological background is $R = 6(2H^2 + \dot{H})$ so that the conformal coupling term

$$\frac{1}{12}R\varphi^2 \simeq H^2\varphi^2\,, \tag{9}$$

strongly resembles the Hubble scale mass term in (2). Due to this similarity the source of the mass term was conjectured to be the conformally coupled scalar in the brane anti-brane inflationary model of [9]. However, this was not explicitly derived. All that was derived was the pure mass term $H^2\varphi^2$, analogous to the famed supergravity (SUGRA), dimension-six operator mass term, $\hat{\mathcal{O}}_6 = V_0\bar{\varphi}\varphi$, that results from expanding an F-term potential and using the Einstein equations in an Friedmann-Robertson-Walker (FRW) background to identify V_0 with the Hubble parameter during inflation H_0. An important point, is that the SUGRA mass term is present *independently* from the conformal term (9)[6].

Besides the conformal term (9), the mass term from the SUGRA analysis [6, 9] and the terms arising due to moduli-stabilization effects [2–4] there are other possible EFT sources of problematic mass terms, for example, finite temperature effects. Typically, finite temperature corrections to the one-loop effective inflaton potential in the high temperature limit give rise to a temperature dependent mass term of the form [23]:

$$\Delta V^T = CT^2\varphi^2\,. \tag{10}$$

Gibbons and Hawking used the path-integral approach to quantum field theory to prove that Green's functions in de Sitter spacetime are periodic in imaginary time and yield the Hawking temperature [24]:

$$T_{dS} = \frac{H}{2\pi}\,. \tag{11}$$

Each observer in de Sitter space has an event horizon, which radiates a thermal spectrum of particles at the temperature (11). Combining (10) with (11) yields a finite temperature effective mass term correction[7] $\sim\ H^2\varphi^2$.

Finally, we mention that it is perfectly consistent to include mass terms from both the non-minimally coupled scalar and various other contributions mentioned above. An interesting example, where the non-minimal coupling term appears in addition to an explicit $H^2\varphi^2$ mass term is in the calculation of the Hubble effective potential at one loop in [25].

Ultimately, the origin of problematic mass terms in (2) is not important for our results pertaining to a successful inflationary trajectory. Although in general there are important distinctions between pure mass terms and the conformal coupling term. For specificity, we consider a contribution from the non-minimally coupled inflaton field parameterizing the D3 brane position as in [9, 21][8]. The 4D effective action is $S = S_{ST} + S_\varphi$, where S_{ST} is the scalar-tensor action composed of the Einstein-Hilbert action plus non-minimally coupled scalar mass term

$$S_{ST} = \int d^4x\,\sqrt{-g}\,\left\{\frac{M_{pl}^2}{2}R - \frac{\xi}{2}R\varphi^2\right\}\,, \tag{12}$$

and S_φ is the (slow-roll limit) canonically normalized inflaton kinetic term plus potential $V(\varphi)$. In this paper, we will consider the conformal value $\xi = 1/6$ and the $\xi = 0$ case corresponding to minimal coupling. The specific form for the potential will not play a significant role in our results.

Varying the action with respect to the metric gives:

$$\begin{aligned}(M_{pl}^2 - \xi\varphi^2)G_{\mu\nu} &= \partial_\mu\varphi\,\partial_\nu\varphi - \frac{1}{2}\partial^\rho\varphi\partial_\rho\varphi\,g_{\mu\nu} \\ &-\xi\Big(\nabla_\mu\nabla_\nu(\varphi^2) - \nabla_\sigma\nabla^\sigma(\varphi^2)g_{\mu\nu}\Big) - V(\varphi)g_{\mu\nu}\quad.\end{aligned} \tag{13}$$

[5] Note, the conformal symmetry is broken by the presence of the potential and by the DBI nature of the kinetic term.

[6] In addition, specific symmetries may force $\xi = 0$. We thank Andrei Linde for discussions of these points.

[7] We thank Henry Tye for discussions of this point.

[8] We thank Juan Maldacena and Shinji Mukohyama for clarifications of this point.

The equation of motion for φ is:

$$\nabla^\mu \nabla_\mu \varphi - V' - \xi R \varphi = 0 \,. \qquad (14)$$

4. NUMERICAL SOLUTIONS

For an observationally favored, flat ($k = 0$), cosmological (FRW) metric ansatz

$$ds^2 = -dt^2 + a^2(t) d\mathbf{x}^2 \,, \qquad (15)$$

the Friedmann equation and equation of motion for φ are:

$$3(M_{pl}^2 - \xi\varphi^2)H^2 = \frac{1}{2}\dot{\varphi}^2 + 6H\xi\varphi\dot{\varphi} + V(\varphi)\,, \qquad (16)$$

$$\ddot{\varphi} + 3H\dot{\varphi} + 6\xi\varphi(2H^2 + \dot{H}) + V' = 0\,, \qquad (17)$$

where $a(t)$ is the scale factor as a function of cosmic time t and $H = \dot{a}/a$ is the Hubble parameter. We consider the inflection point potential (8). In the following solutions we adopt the parameter values $\lambda_1 = 7 \times 10^{-5}$, $\lambda_2 = 60$ of [26], chosen to produce the correct normalization of density perturbations, although in this example we take $V_0 = 10^{-8}$ corresponding to a GUT inflationary scale $M \simeq V_0^{1/4} \simeq 10^{16}\,\text{GeV}$ and $\varphi_\star = .1$. All quantities are given in Planck units. We emphasize that these particular numbers are not essential to our general findings. The potential is plotted in Fig.1. The throat is supported by

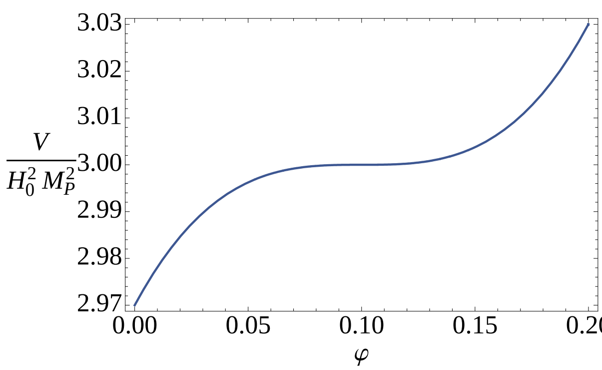

FIG. 1: Inflection point potential (8).

N dissolved D3-branes; in terms of a re-scaled λ parameter of (6), $\bar{\lambda} \equiv \lambda(H_0/M_{pl})^2$ and the inflationary scale M:

$$N = 6\pi^2 \bar{\lambda} \left(\frac{M_{pl}}{M}\right)^4 \,, \qquad (18)$$

where we have defined

$$H_0^2 \equiv \frac{V_0}{3M_{pl}^2} = \frac{1}{3}\frac{M^4}{M_{pl}^2}\,. \qquad (19)$$

For the above compactification data, $N = 60$, giving a BM bound (7) of $\Delta\varphi_{max} = .26$. In our examples we take the initial value of the inflaton to be well within this limit, $\varphi_i = .15$ and near the inflection point.

Using the above data it is possible to build a slow-roll inflationary model with $\mathcal{N} \geq 60$ e–foldings near the inflection point of (8) if the inflaton is minimally coupled ($\xi = 0$). We provide a corresponding numerical solution in Fig. 2. The solution is an example of a successful slow-roll inflationary solution discussed in [3].

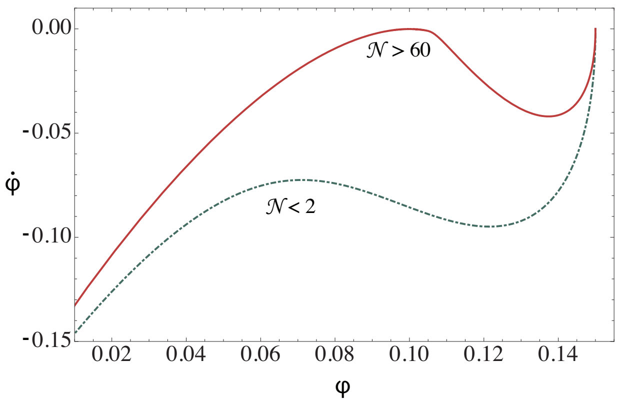

FIG. 2: Phase portrait showing the inflaton trajectory in the inflection point potential for (a) minimally coupled inflaton (solid red) and conformally coupled inflaton (dashed gray).

The situation changes dramatically, however, if we include the effects of the conformal ($\xi = 1/6$) coupling term in (12). Before discussing the altered solution, let us take a moment to ascertain some of the general consequences of including the ξ-term. First, the mass term introduces an effective Planck Mass:

$$M_{\text{eff}}^2 \equiv M_{pl}^2 - \xi\varphi^2 \,. \qquad (20)$$

For a sensical gravitational theory we demand this remain positive, $M_{\text{eff}}^2 > 0$ [27]. For the case $\xi > 0$, this introduces an effective UV cutoff for the theory, at the critical value

$$\frac{\varphi_c}{M_{pl}} = \frac{1}{\sqrt{\xi}}\,. \qquad (21)$$

Thus, the introduction of the ξ-term can destroy large field models of inflation in general. This cutoff exists independently from the field range bound (7). In fact, the bound (21) is more general than the BM bound which holds only in the specific instance of the warped throat brane inflation scenario. In a typical setting, $N > 4\xi$, so that the BM bound, if applicable, is the more restrictive of the two. Finally, we mention this cutoff will have strong implications for models of inflation where φ is growing, and hence, approaching the critical value. Such is the case in infra-red (IR) models of brane inflation [28].

Secondly, we can think of the conformal coupling term as generating an effective potential (see (17)):

$$V'_{\text{eff}}(\varphi) = \varphi(2H^2 + \dot{H}) + V'\,, \qquad (22)$$

effectively steepening the potential, decreasing the possibility for slow-roll inflation. This effect has noteworthy implications for the existence of eternal inflation [29]

in the brane anti-brane model of [9], or other models with very flat potentials. Regions of the Universe where eternal inflation can occur have an indefinitely large and growing volume, making these regions statistically favored [30]. For inflating solutions with very flat potentials, $\dot\varphi \simeq \dot H \simeq V' \simeq 0$, (17) implies the conformal term generates a new downward force $\ddot\varphi \simeq -2\,H^2\varphi$ typically decreasing the number of favorable regions where eternal inflation can occur.

In the numerical analysis plotted in Fig. 2 we see that including the conformally coupled scalar allows for only $\mathcal{N} \simeq 2$ e–foldings of inflation (although there may be an interesting possibility of rapid-roll inflation at sufficiently low inflationary scales, see [31]).

Using the same compactification data and initial conditions as above, we now appropriately take into account the non-standard DBI form of the kinetic term for the inflaton field. Thus, the entire action for the system is (12) added to

$$\frac{\mathcal{L}_{DBI}}{\sqrt{-g}} = -\frac{1}{f(\varphi)}\left[\sqrt{1+f(\varphi)\,g^{\mu\nu}\,\partial_\mu\varphi\,\partial_\nu\varphi}-1\right], \quad (23)$$

and potential (8).

Variation with respect to the metric $g^{\mu\nu}$ leads to:

$$(M_{pl}^2-\xi\varphi^2)G_{\mu\nu} = \frac{\partial_\mu\varphi\,\partial_\nu\varphi - g_{\mu\nu}\,\left(\partial^\sigma\varphi\,\partial_\sigma\varphi + f^{-1}(\varphi)\right)}{\sqrt{1+f(\varphi)(\partial\varphi)^2}} - \xi\Big(\nabla_\mu\nabla_\nu(\varphi^2) - \nabla_\sigma\nabla^\sigma(\varphi^2)g_{\mu\nu}\Big) + \left(f^{-1}(\varphi) - V(\varphi)\right)g_{\mu\nu}\,. \quad (24)$$

The equation of motion for the field φ is:

$$\nabla_\mu(\gamma\,\partial^\mu\varphi) + f^{-2}f'(\gamma^{-1}-1) - \frac{1}{2}f^{-1}f'\,\gamma\,(\partial\varphi)^2 - V' - \xi R\varphi = 0\,,$$

where

$$\gamma \equiv \frac{1}{\sqrt{1+f(\varphi)(\partial\varphi)^2}}\,, \quad (25)$$

and prime denotes differentiation with respect to φ. The resulting Friedmann equation and equation of motion for φ are:

$$\begin{aligned} 3\left(M_{pl}^2 - \frac{\varphi^2}{6}\right)H^2 &= f^{-1}(\gamma-1) + H\varphi\dot\varphi + V(\varphi)\,, \quad (26)\\ \gamma^3\ddot\varphi + 3H\gamma\dot\varphi &+ V' + \varphi(2H^2+\dot H)\\ &+ \frac{f'}{2f^2}\left((3f\dot\varphi^2-2)\gamma^3+2\right) = 0\,, \quad (27) \end{aligned}$$

respectively, where $\gamma = (1-f\dot\varphi^2)^{-1/2}$. Solving the equations we find that taking into account the nonstandard nature of the kinetic term restores a successful inflationary model leading to more than 60 e–foldings of relativistic DBI inflation, circumventing the eta problem (Fig. 3). While this result is not entirely unexpected it has not yet explicitly been shown in the literature in the context of a conformally coupled inflaton.

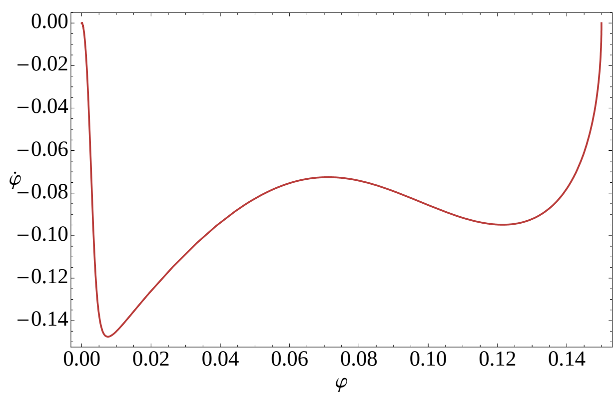

FIG. 3: Phase portrait for inflaton with DBI kinetic term and $\xi = 1/6$. $\mathcal{N} \geq 60$ e–foldings are achieved.

5. OBSERVATIONAL PREDICTIONS

As we have already mentioned, it is not our goal to fit the above model to the latest observational data, but rather to present a solution that avoids the eta problem and successfully produces more than 60 e–foldings of inflation while simultaneously satisfying all known compactification bounds. However, despite our current modest goals, it is interesting to understand in generality the possible effects of gravitational couplings in theories with nonstandard kinetic terms on cosmological observables and we initiate this study here.

In this Section we show that the presence of a nonminmally coupled inflaton field in a DBI action changes the overall behavior of the sound speed in the model. In principle, this can lead to significant observational effects, for example, in the form of non-gaussianities in the CMB, allowing for possible observational distinction between pure DBI and DBI with non-minimally coupled inflaton[9].

[9] CMB constraints on kinetically modified inflation models were

To calculate the observable quantities from the action (12) coupled to (23) we first perform the weyl rescaling:

$$\tilde{g}_{\mu\nu} = \Omega^2(\varphi) g_{\mu\nu}\,, \tag{28}$$

where Ω^2 is the scaling factor given in terms of the inflaton as

$$\Omega^2 = 1 - \xi \frac{\varphi^2}{M_{pl}^2}\,. \tag{29}$$

Under the transformation the action (23) becomes

$$S = \int d^4x\, \sqrt{-\tilde{g}} \left(\frac{M_{pl}^2}{2} \tilde{R} + P(X, \varphi) \right)\,, \tag{30}$$

where $X \equiv -\frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$. The functional form for P is given by:

$$\left(1 - \xi\frac{\varphi^2}{M_{pl}^2}\right)^2 P(\varphi, X) = 6\xi^2 X \frac{\varphi^2}{M_{pl}^2} \tag{31}$$
$$-f^{-1}(\varphi)\sqrt{1 - 2Xf\left(1 - \xi\frac{\varphi^2}{M_{pl}^2}\right)} \;+\; f^{-1}(\varphi) - V(\varphi)\,.$$

We denote this frame as the *P-frame*. Note that this *is not* the traditional Einstein frame, since in the low energy limit the action for φ does not reduce to that of a cannonically normalized scalar field as it does in the special case where P is that of ordinary DBI inflation. We now calculate the physical observables directly in the P frame [34]. The theory of perturbations for actions of the form (30) was carried out in [35]. We will follow the study of primordial scalar non-gaussiantities in [36].

The inflaton energy is given by:

$$E = 2XP_{,X} - P\,, \tag{32}$$

and quantities of measurable interest are conveniently written in terms of the generalized slow-roll parameters:

$$\begin{aligned} \epsilon &= -\frac{\dot{H}}{H^2} = \frac{XP_{,X}}{M_{pl}^2 H^2}\,, \\ \eta &= \frac{\dot{\epsilon}}{\epsilon H}\,, \\ s &= \frac{\dot{c}_s}{c_s H}\,, \end{aligned} \tag{33}$$

where c_s is the sound speed:

$$c_s^2 = \frac{dP}{dE} = \frac{P_{,X}}{P_{,X} + 2XP_{,XX}}\,, \tag{34}$$

considered in, e.g., [32].

which is most conveniently calculated in terms of the rescaled warp factor:

$$\tilde{f}(\varphi) \equiv f(\varphi)\left(1 - \xi\frac{\varphi^2}{M_{pl}^2}\right)\,. \tag{35}$$

The result for our $P(X, \varphi)$ (31) is:

$$c_s^2 = \frac{(1 - \tilde{f}\dot{\varphi}^2)(1 - \xi\frac{\varphi^2}{M_{pl}^2} + 6\xi^2\sqrt{1 - \tilde{f}\dot{\varphi}^2}\frac{\varphi^2}{M_{pl}^2})}{1 - \xi\frac{\varphi^2}{M_{pl}^2} + 6\xi^2(1 - \tilde{f}\dot{\varphi}^2)^{3/2}\frac{\varphi^2}{M_{pl}^2}}\,. \tag{36}$$

In the limit of non-minimal coupling, $\xi = 0$, (36) reduces to the standard result for DBI inflation:

$$c_{s,\,DBI}^2 = 1 - f\dot{\varphi}^2\,. \tag{37}$$

However, for the case under consideration, the results are slightly modified by the scalar-gravitational coupling term, allowing, at least in principle, for observationally distinct signatures. The observable quantities of interest include, the primordial power spectrum:

$$P_k^\zeta = \frac{1}{8\pi^2 M_{pl}^2}\frac{H^2}{c_s\epsilon}\,, \tag{38}$$

the tensor perturbation spectrum:

$$P_k^h = \frac{2}{3\pi^2}\frac{E}{M_{pl}^4}\,, \tag{39}$$

and their respective spectral indices n_s and n_T. In this paper, we choose to focus on observational signatures in the form of deviations from gaussianity of the CMB. Non-gaussianities are sensitive to the three point correlation function of the Fourier transform of the gauge invariant curvature perturbation ζ [37]. A useful way to quantify the level of non-gaussianity in a given model is in terms of the scalar quantity, f_{NL}, given by

$$\zeta = \zeta_L - \frac{3}{5} f_{NL}\zeta_L^2\,, \tag{40}$$

where ζ_L is the linear gaussian part of the perturbation[10]. Single field, slow-roll models generically predict undetectable levels of non-gaussianity [38]. This precise form for f_{NL} is valid for the *local* form of non-gaussianity. For the P-frame action (30):

$$f_{NL} = \frac{5}{81}\left(1 - \frac{1}{c_s^2} + 2\Lambda\right) - \frac{35}{108}\left(1 - \frac{1}{c_s^2}\right)\,, \tag{41}$$

where

$$\Lambda \equiv \frac{X^2 P_{,XX} + \frac{2}{3}X^3 P_{,XXX}}{XP_{,X} + 2X^2 P_{,XX}}\,. \tag{42}$$

[10] Here we use the sign conventions for f_{NL} of [36].

For the case of pure DBI, with sound speed (37), the first term in (41) vanishes [7, 33]. However, in the case of DBI plus non-minimally coupled scalar this is no longer the case (the second term is also altered with respect to the pure DBI form via the sound speed modification). The overall change in the sound speed due to the presence of the non-minimal coupling term opens up the possibility that non-gaussianity and other observables listed above may be used to probe gravitational couplings of the form (9) (or other such terms). While we have focused on the DBI case above, we expect similar behavior in any model that contributes measurable non-gaussianities such as the K-inflation models of [35].

6. CONCLUSIONS

In this paper we have explicitly demonstrated the ability of the DBI mechanism to circumvent the eta problem in a class of string inflation models. We have worked within the context of a non-minimally coupled inflaton field, however, our results apply to more general situations of pure Hubble mass corrections to the inflaton potential. Sources of Hubble mass corrections, both in the context of effective field theory and in the context of string theory were discussed in Section 3.

DBI inflation models typically face a particular challenge due to backreaction effects and we have nothing new to add to this discussion here [5, 15, 39]. Indeed, the sound speed in the above solution grows very small rapidly and produces large non-gaussianity in excess of current observational bounds. Nevertheless, our goal was to obtain an inflationary solution that makes progress towards realizing inflation even with Hubble mass inflaton. In the present context it is possible to construct tuned inflaton trajectories that are initially slow-roll and fall into the DBI regime only towards the end of inflation. Because of this, f_{NL} can become large after the CMB scales set in. This leaves the possibility of a model gaining the majority of e–foldings from standard slow-roll and the rest from DBI with detectable, yet observationally acceptable, levels of non-gaussianity. We have not determined wether such solutions are compatible with the stringy compactification constraints of [17].

Finally, we have pointed out that, because adding a non-minimally coupled scalar to the DBI or K-inflation actions changes the formula for the speed of sound (and observable quantities), gravitational couplings can contribute potentially observable signatures in the CMB. We leave a more detailed analysis of this interesting possibility for future work [40].

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